Name : ....

Third Semester B.Tech. Degree Examination, January 2016
(2013 Scheme)
13.301 : ENGINEERING MATHEMATICS – II
(ABCEFHMNPRSTU)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 100

PART – A

(Answer **all** questions. **Each** question carries **4** marks.)

1. Find the directional derivative of $\phi = x^2 + xy + z^2$ at (1, –1, –1) in a direction towards the point (3, 2, 1).

2. Find the sine transform of $f(x) = \sin x$ in $0 < x < \pi$.

3. Obtain the partial differential equation by eliminating the arbitrary function from

$$z = f(y + 3x) + g(y - 3x) + \frac{1}{6}x^3y.$$

4. Find the particular integral of $\left(D^2 + 2DD' + D'^2\right)z = e^{x-y}$.

5. Using the method of separation of variables, solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, given that $u(x, 0) = 4e^{-x}$.

P.T.O.

PART – B

(Answer **one full** question from **each** Module. **Each** question carries **20** marks).

MODULE – I

6. a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$ where $\vec{r} = xi + yj + zk$ and $r = |\vec{r}|$.

   b) Find the work done by the force $\vec{F} = (yz + 2x)i + xzj + (xy + 2z)k$ when it moves a particle along the curve $x^2 + y^2 = 1$, $z = 1$ in the positive direction from (0, 1, 1) to (1, 0, 1).

   c) Use divergence theorem to evaluate $\iint_S \vec{A}.\vec{dS}$ where $\vec{A} = 12x^2yi - 3yzj + 2zk$ and S is the portion of the plane $x + y + z = 1$ included in the first octant.

7. a) Show that $\vec{F} = (y^2 - 2xyz^3)i + (3 + 2xy - x^2z^3)j + (6z^3 - 3x^2yz^2)k$ is irrotational and hence find its scalar potential.

   b) Use Green's theorem in a plane to evaluate $\int_C \left[(2x - y^3)dx - xydy\right]$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

   c) Evaluate $\iint_S \text{curl}\,\vec{F} \cdot \hat{n}\,dS$ using Stoke's theorem where S is the part of the surface of the paraboloid $x^2 + y^2 + z = 1$ for which $z \geq 0$ and $\vec{F} = yi + zj + xk$.

MODULE – II

8. a) Obtain the Fourier series of $f(x) = 2x - x^2$ in $0 < x < 3$.

   b) Find the half range cosine series of $f(x) = x\sin x$ in $0 < x < \pi$.

   c) Find the Fourier transform of $f(x) = \begin{cases} \cos x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.

9. a) Obtain the Fourier series of $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(x - 2), & 1 < x \leq 2 \end{cases}$.

   b) Obtain the half range sine series of $f(x) = e^x$ in $0 < x < \pi$.

   c) Find the Fourier cosine transform of $f(x) = e^{-5x}$.

MODULE – III

10. a) Solve $p^2q^2 + x^2y^2 = x^2q^2(x^2 + y^2)$.

b) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$.

c) Solve $\left(D^3 - 7DD'^2 - 6D'^3\right)z = \sin(x + 2y)$.

11. a) Solve by Charpit's method, $(p^2 + q^2)x = pz$.

b) Solve $\frac{p}{x^2} + \frac{q}{y^2} = z$.

c) Solve $\left(D^2 - 2DD' + D'^2\right)z = e^x(x + 2y)$.

MODULE – IV

12. a) If a string of length 'l' is initially at rest in the equilibrium position and each of its points is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = V_0 \sin^3\left(\frac{\pi x}{l}\right), 0 < x < l$. Find the displacement function y(x, t).

b) The ends A and B of a rod 20 cms long have the temperature at 0° C and 80° C until steady conditions prevail. If the temperature at B is reduced to 0° C and kept so while that of A is maintained, find the temperature function u(x, t).

13. a) A uniform elastic string of length 60 cm is subjected to a constant tension of 2 kg. If the ends are fixed and the initial displacement $y(x, 0) = 60x - x^2, 0 < x < 60$, while the initial velocity is zero, find the displacement function y(x, t).

b) Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ under the conditions

i) $\frac{\partial u}{\partial x}(0, t) = 0$ for $t \geq 0$

ii) $\frac{\partial u}{\partial x}(\pi, t) = 0$ for $t \geq 0$

iii) $u(x, 0) = x^2, 0 < x < \pi$.
